Chemistry 215 B
Homework # 4

1) Sakurai Problem 5-29.

2) Consider the Jaynes-Cumming Hamilitonian

$$H = \frac{1}{2}\epsilon\hat{\sigma}_z + \hbar\omega_0\hat{a}^\dagger\hat{a} + g\left(\hat{a}^\dagger\hat{\sigma}_- + \hat{a}\hat{\sigma}_+\right) \tag{1}$$

where, as usual, the $\hat{\sigma}_z, \hat{\sigma}_\pm$ are the Pauli z matrix and the raising and lowering spin-1/2 operators. The first term of the Hamiltonian represents just a two-level system with energies $\pm\epsilon$. The operators $\hat{a}^\dagger$ and $\hat{a}$ are raising and lowering operators for a harmonic oscillator. The second term represents the Hamiltonian of this part of the system. The c-number g couples these two systems.

A bit of background: This basic Hamiltonian is used in a wide variety of systems. Some examples include a microscopic cantilever coupled to a quantum dot where electrons can hop between the dot and cantilever. The two-level system represents the states of the dot with 0 or 1 electronic charge and the harmonic oscillator the (dominant) elastic mode of the cantilever. Another example comes from the interaction of an atom in a cavity where the atom can be in one of two atomic levels (the two-state system) and transfer energy to the fundamental radiation mode of the cavity. This background will not help you solve the problem, but should make working on it more fun. You can do a quick search on the Googles to see more instances where this Hamiltonian makes an appearance.

a) The decoupled system: Write down the spectrum and eigenkets (in our usual Dirac notation) for the decoupled system, i.e. where $g = 0$. Draw each state as a horizontal line segment with an energy axis running vertically on the page. Be sure to label each state with the appropriate quantum numbers.

b) Weak coupling: Assuming that $\epsilon - \hbar\omega_0 \neq 0$, find the effect of a finite coupling g on the spectrum you computed in part a to the lowest nontrivial order in g. In other words you have to go far enough in perturbation theory to see an effect, but you don't need to compute the perturbative correction the effect that you found.

c) You find the oscillator in the n^{th} excited state and the two-level system in the lower energy state. Use time-dependent perturbation theory to compute the transition rate to a state with one less quantum of energy in the oscillator but the two-level system in the higher energy state. In other words calculate the transition probability of energy from the oscillator to the two-level system. Also compute the transition rate of the two-level system from the upper to the lower state. Here the oscillator gains energy. The first is an absorption process and the second is called stimulated emission.